Stochastic Tools Homework 4

Stochastic Tools in Turbulence, AEM-ADV18
Instructor: William K. George, 2013-03-12

Refence material: Chapter 8 and Appendices B through F of WKG's 'Lectures in Turbulence for the 21st Century.

Access the data set you were given in assignment 2 and carry out the analysis described below.

1. Mean and variance (Repeated from assignment 2)
   (a) Compute the mean and variance using all blocks of data.
   (b) Now subdivide the data into ten parts and compute the mean and variance of each part separately. These probably will not be exactly the same, but you will be asked to discuss this below.
2. Spectral Analysis
   (a) Compute the frequency (circular) spectrum (using the FFT and your knowledge of Fortan, etc). Show both linear-linear and log-log plots of your results. Do this three ways: first with just one block of data, then with 10 blocks of data, and finally with all the data. Do you notice any differences if you do not use blocks which have an integer power of 2 (e.g., speed of computation, spurious peaks appearing, etc.)
   (b) The variability for a spectral estimator is just given by $2/N$ where N is the number of independent estimates at each frequency. (Note that it is independent of frequency.) Compute the variability for your spectral estimator and use it to place error bounds on your spectral estimates at different frequencies. Note how different these look for log-log and lin-lin plots. Why?
   (c) Compare the integral under the spectrum with the variance computed in above. Try to explain or eliminate any discrepancies (hint: Look for obvious evidence of noise. And don't forget you should be working with a whole line spectrum so only half of the energy is at postive frequencies. Where are the negative ones hiding in your output?). If different which should you believe to be the most reliable.
   (d) Now use the your spectrum and variance to estimate the integral scale (using the intercept as $f \to 0$). How accurate would you guess this estimate to be (based on the accuracy of your variance and spectrum and extrapolation)?
3. Correlation Analysis

(a) Compute the two-point correlation directly from the data by averaging the products of two velocities at different times (You were supposed to have done this last time).

(b) Estimate the statistical error for several time lags and show them on your plot.

(c) Now estimate the integral time scale by integrating under the autocorrelation function, and compare the integral scale obtained in this manner to the one you estimated from the spectra above using the zero frequency intercept. Which do you believe to be the most accurate? Why?

(d) Now compute the autocorrelation by taking the inverse Fourier transform of the spectrum you computed above (Remember where the negative frequencies are). Plot them together and explain any differences. If different, which should you believe, if either. Explain why you believe this.